

HIGH SCHOOL STUDENTS' ONTOGENIC OBSTACLES IN SOLVING SOLID GEOMETRY PROBLEMS VIEWED FROM SPATIAL ABILITY

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Abstract: Many students have difficulties understanding the topic of solid geometry due to their limited knowledge, commonly called ontogenic obstacles. This study aims to investigate what ontogenic obstacles arise when students solve solid geometry problems based on spatial ability indicators. A qualitative with case study approach was applied in this study. Participants of this study were 44 twelve graders in one of the public high schools in Serang, Indonesia. Data collection was carried out through a test consisting of 3 questions related to prerequisite material and 3 questions on solid geometry problems based on spatial ability indicators, observations and interviews with students. The results show that ontogenic obstacles found in this study include psychological, instrumental, and conceptual ontogenic obstacles. First is psychological ontogenic obstacles that occur when students do not like mathematics, so they easily forget the material they have learned. Second is instrumental ontogenic obstacles that occur when students are less prepared in understanding the distance concepts of solid geometry such as distance between point-to-point, point-to-line, and point-to-plane so students are mistaken in solving solid geometry problems based on spatial ability indicators. Third is conceptual ontogenic obstacles that occur when some students do not understand arithmetic operations, so they are confused in solving arithmetic operations related to the problem. Then overall, the ontogenic obstacles that are most often experienced by students include constructing solid figures into the appropriate plane figures, constructing plane figures that are as expected, and completing arithmetic operations.

Keywords: Ontogenic obstacles, Solid geometry, Spatial ability

Introduction

Mathematics is one of the subjects that train students to have the ability to think logically, analytically, systematically, critically, and creatively. Cockcroft (1982) states that mathematics needs to be taught because (1) mathematics will be useful for various aspects of life, all studies require mathematical knowledge by these studies; (2) mathematics is a strong, concise, and clear means of communication; (3) mathematics can also be used to present information in various ways; (4) mathematics can improve logical thinking skills, accuracy, and spatial awareness; (5) mathematics can also provide a sense of satisfaction in solving a problem. Therefore, mathematics has a very important role in everyday life.

One of the branches of mathematics that contains the concepts of point, line, plane, and space is geometry. Geometry is not only prominent in deductive methods and abstract objects but also an effective technique in solving mathematical problems (Nurjanah et al., 2014). The National Council of

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Teachers of Mathematics (NCTM, 2000) revealed that one of the standards for teaching geometry in schools is to invite students to analyze the characteristics of geometric shapes and make mathematical arguments about geometric relationships, as well as to get them to use visualization, spatial sense, and geometric modelling to solve problems. In line with NCTM's opinion, the curriculum in Indonesia requires children to master plane geometry and space geometry material in which there are also spatial abilities (Siswanto & Kusumah, 2017). Nurjanah et al. (2014) revealed that the National Curriculum in Indonesia places geometry as a very important aspect of learning mathematics because it exists in everyday life and includes spatial objects. This shows that the importance of learning geometry is because geometry is related to spatial ability.

Spatial ability is an ability related to visual imagery. Nurjanah et al. (2014) stated that spatial ability is an intuitive awareness of shape and space that requires an understanding of geometry concepts as well as the ability to understand, visualize, describe, and see objects from different perspectives and transform geometric shapes. According to McGee (1979), "spatial ability consists of spatial skills as changing, rotating, bending and reversing of an object presented for stimulating in the mind". Linn & Petersen (1985) defined spatial ability as a mental process used in sensing, storing, recalling, creating, composing, and making relationships about spatial shapes. Maier (1996) states that there are five elements of spatial ability: spatial perception, visualization, mental rotation, spatial relation, and spatial orientation. In essence, spatial ability is the ability to visualize a geometric object in the mind of an individual.

Several studies reveal that spatial ability has a very important role in geometry learning and everyday life. Yilmaz (2009) stated that spatial ability is a comprehensive construct that has an impact on a person's daily life, school achievement, and success in certain types of work. Furthermore, research by Guzel & Sener (2009) states that spatial ability helps students in understanding images easily, comment on visualized information, create context between various concepts easily, generalize complex concepts, and think in different ways. Not only that, according to Rodán et al. (2019) spatial abilities are very important in different daily tasks, such as driving, following instructions to collect pieces of furniture, or orientating a given space. Meanwhile, according to Nurjanah et al. (2014), spatial ability is one of the factors that have contributed to geometry learning, in this case, strong spatial relationships and competence in utilizing geometry concepts and geometry language can improve students' understanding of measurement and numbers. Therefore, spatial ability has an important role in success in learning mathematics, especially learning geometry.

Seeing how important geometry learning is, it should make geometry one of the things that are considered. But in reality, according to Nurjanah et al. (2017), geometry as one of the branches of mathematics is currently in a concerning position. Furthermore, Nurjanah et al. (2014) stated that most students find it difficult to understand geometry, especially spatial geometry. The results of research by Šipuš & Čizmešija (2012) stated that the most frequent error based on their research on the Spatial ability of students of mathematics education in Croatia evaluated by the Mental Cutting Test is that students do not recognize the spatial form of the object given. Then the results of research by Güven & Kosa (2008) revealed that two important factors cause low spatial abilities of students including (1) one of the reasons for low scores is presenting three-dimensional spatial information in a two-dimensional format on the blackboard in traditional geometry lessons in Turkey, because of this limitation students do not have the opportunity to create and manipulate three-dimensional models that have vital

importance for developing spatial skills; (2) because university entrance exams use multiple choice, geometry learning in Turkey is largely based on procedural teaching. This suggests that low geometry ability has something to do with the low spatial ability of students.

The low spatial ability of students certainly cannot be separated from the term learning obstacles. According to Brousseau (2002), students naturally experience a situation called learning obstacles, while learning obstacles are divided into three categories: ontogenic obstacles, epistemological obstacles, and didactical obstacles. Learning obstacles are caused by two factors, namely internal and external factors. Internal obstacles are obstacles that come from within the students themselves, while external obstacles are things that come from the environment around students. The existence of these learning obstacles can affect students' ability to solve a problem.

Based on the explanation above, researchers are interested in examining the learning obstacles experienced by class XII students, especially ontogenic obstacles in solving solid geometry problems, especially distance concepts material, especially cubes and beams based on elements of spatial ability. The formulation of the problem studied in this study is how ontogenic obstacles are experienced by class XII students in solving solid geometry problems (distance concepts of solid geometry, especially in cubes and blocks) based on the elements of spatial ability.

Materials and Methods

Materials

Learning Obstacles

The obstacle itself means obstacle; obstacle. So it can be said that learning obstacles are obstacles or obstacles that cause students to be slow in the process of changing behaviour. Learning obstacles are caused by two factors, namely internal and external factors. Internal obstacles are obstacles that come from within the students themselves. Meanwhile, external obstacles are things that come from the environment around students. According to Brousseau (2002), students naturally experience situations called learning obstacles, learning obstacles are divided into three categories: ontogenic obstacles, epistemological obstacles, and didactical obstacles.

Ontogenic obstacles are obstacles that occur due to student limitations (Brousseau, 2002). According to Suryadi (2019), ontogenic obstacles are divided into three types, namely:

1. Ontogenic psychological obstacles, are student unpreparedness related to motivation and interest in the material being studied;
2. Ontogenic instrumental obstacles, are student unpreparedness related to key technical matters of a learning process, which can be revealed, for example, through responses and errors in the student's completion process;
3. Ontogenic conceptual obstacles are student unpreparedness related to previous learning experiences, such as a lack of mastery of basic concepts and prerequisites supporting the material.

Solid Geometry

Solid geometry is one of the materials in geometry that discusses space. The solid geometry material studied in this research is the distance concepts of solid geometry, which includes distance between point-to-point, point-to-line, and point-to-plane.

Point-to-point distance

The distance between two points is the length of the line segment that connects the two points. To determine the distance from point A to point B in a space can be done by connecting point A and point B so that it becomes a line segment AB. The length of the line segment AB is the shortest distance between point A and point B.

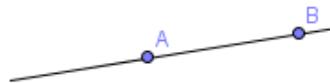


Figure 1: *Point-to-point distance.*

Point-to-line distance

The distance between point A and line g with A not lying on line g is the length of the line segment drawn from point A and perpendicular to line g.

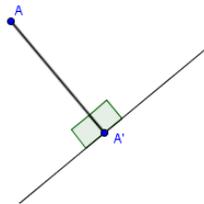


Figure 2: *Point-to-line distance.*

The steps in determining the distance from point A to line g (point A does not lie on line g) are as follows.

1. Create a line segment AA' perpendicular to line g in the α -plane;
2. The length of line segment AA' is the shortest distance from point A to line g.

Point-to-plane distance

The distance between point A and plane α (A does not lie on plane α) is the length of the perpendicular line segment from point A to plane α .

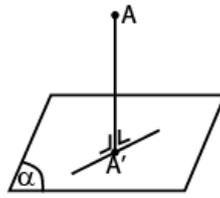


Figure 3: *Point-to-plane distance*

The steps in determining the distance of point A to plane α (point A does not lie on plane α) are as follows.

1. Draw a line g through point A and perpendicular to plane α ;
2. Line g penetrates plane α at point A' ;
3. The length of line segment AA' is the distance from point A to plane α .

Spatial Ability

According to Mcgee (1979), "spatial ability consists of spatial skills as changing, rotating, bending and reversing of an object presented for stimulating in the mind". Linn & Petersen (1985) defined spatial ability as a mental process used in sensing, storing, recalling, creating, composing, and making relationships about spatial structures. Furthermore, Nurjanah et al. (2014) revealed that spatial ability is an intuitive awareness of shape and space that requires understanding geometry concepts and understanding, visualising, describing, and seeing objects from different perspectives and transforming geometric shapes.

Maier (1996) states that there are five elements of spatial ability: spatial perception, visualization, mental rotation, spatial relation, and spatial orientation. The following is an explanation of each element of spatial ability according to Maier (1996)

1. Spatial Perception, spatial perception tests require the location of the horizontal or the vertical despite distracting information;
2. Visualization is the ability to visualize configurations where there is movement or displacement between (internal) parts of the configuration;
3. Mental Rotation is the ability to rotate plane figures or solid figures quickly and accurately;
4. Spatial Relation is the ability to understand the spatial configuration of objects or parts of an object and their relationship to each other;
5. Spatial Orientation is the ability to orientate oneself physically or mentally in a space that requires one's orientation in a particular matter.

The indicators of spatial ability used in this study are indicators of spatial ability according to Sutadnyana (2013), namely.

1. Determining the position of an object;
2. Accurately mentioning the actual figures of solid geometry viewed from a certain point of view;
3. Determining the actual size of an object's visual stimulus.;
4. Construct and represent geometry models drawn on plane figures.

Methods

The participants in this study were 44 grade XII students in one of the public high schools in Serang City, Banten Province who had studied the distance concept of solid geometry material. This research uses a qualitative with a case study approach where the researcher is the key instrument in this research. Data collection in this study consisted of test instruments, observation, and interviews. The test instrument used in this research aims to find out how students' abilities in solving solid geometry problems are based on spatial ability indicators. This written test totalled 6 items in the form of descriptions, with questions number one to three related to the Pythagorean Theorem which is prerequisite material, while questions number four to six are solid geometry problems. The questions were arranged based on spatial ability indicators. The indicators of spatial ability per item are explained in the following table.

Table 1: Table of Spatial Ability Indicators per Question Item.

No.	Spatial Ability Indicator	Question Number
1	Determining the position of an object.	1a, 1b, 2a, 2b, 4a, 5a, and 6a
2	Accurately mentioning the actual figures of solid geometry viewed from a certain point of view.	4a and 6b
3	Determining the actual size of an object's visual stimulus.	3, 4b, 4c, 5b, 5c, and 6c
4	Construct and represent geometry models drawn on plane figures.	4a, 5a, and 6a

Then the interview aims to strengthen or convince the findings obtained from the test results of the test instrument related to the learning obstacles experienced by students and their reasons. The results of the instrument test were then analyzed so that they could describe the learning obstacles experienced by students. The interviews involved several students who were selected to explore how ontogenic obstacles students experience in solving solid geometry problems based on spatial ability indicators.

After collecting data, data reduction was then carried out by classifying the data according to the obstacles experienced by students. The data presented is based on the test results of test instruments, observations and interviews that show students' ontogenic obstacles when solving solid geometry problems based on spatial ability indicators. The conclusions drawn are based on the entire research process. Then the conclusion is verified, so that the conclusion drawn becomes strong.

Results and Discussion

Based on the four indicators of spatial ability according to Sutadnyana (2013) used in this study, the 44 students who were given the written test had a variety of answers. The researcher made a percentage of students' correct answers for each item to make it easier to see how many students were able to answer the questions correctly. The percentage of students' correct answers for each item per indicator is as follows.

Table 2: Table of Percentage of Student Answers per Question.

No.	Spatial Ability Indicator	Question Number	Percentage of Correct Answers
1	Determining the position of an object.	1a	100%
		1b	100%
		2a	100%
		2b	98%
		4a	68%
		5a	98%
		6a	82%
2	Accurately mentioning the actual figures of solid geometry viewed from a certain point of view.	4a	68%
		6b	93%
3	Determining the actual size of an object's visual stimulus.	3	100%
		4b	98%
		4c	84%
		5b	93%
		5c	79%
		6c	93%
4	Construct and represent geometry models drawn on plane figures.	4a	68%
		5a	98%
		6a	82%

The results of this study indicate that overall students already understand the prerequisite material for distance concepts of solid geometry. This is evidenced by the fact that students did not experience any obstacles when working on questions number 1a, 1b, 2a, 2b, and 3.

Furthermore, for question number 4a, students began to experience obstacles. One student experienced obstacles due to confusion with the square and rectangular figures, one student experienced obstacles in illustrating the intended solid figures, and even one student had a different point of view from other students. As for one of the other students experiencing obstacles, because they have not been able to

illustrate the top view and left view properly, this is shown by the drawings they made, even clearer with the use of numbers on the sides of the drawings made. The left view is rectangular in size but is depicted as a square flat, and vice versa with the top view depicted as a rectangular flat, which should be square because it has the same side size. This is also supported by Nurjanah (2014) who states that there are still many students who have difficulty in learning geometry, one of which is the difficulty of students in recognizing and understanding solid figures and their elements.

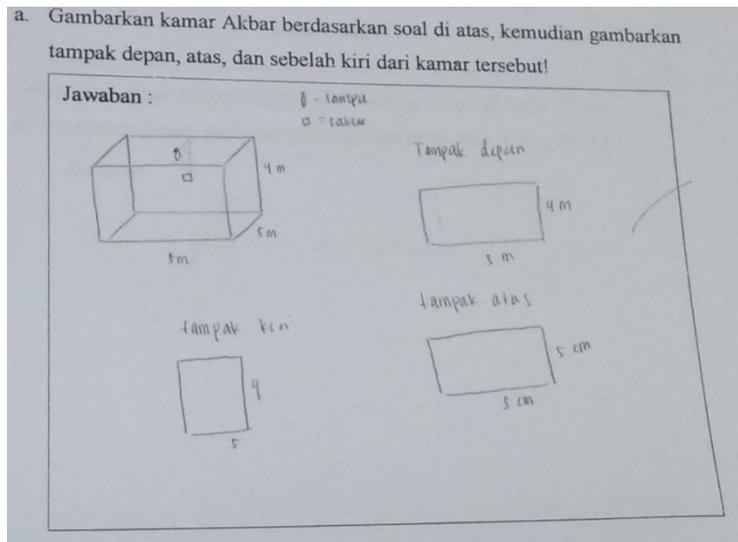


Figure 4. Student Answers Related to Problem Number 4a.

Based on the explanation above, some students still cannot construct plane figures from solid figures. In addition, some students have not been able to visualize the problem properly.

Then for questions number 4b and 4c, 7 students experienced obstacles, one of which was an obstacle in completing algebraic operations. As for problem number 5a, one of the students experienced obstacles in determining the distance between points to the line. This is evidenced by the illustration of the picture he made, it appears that the student shows that the distance from point V to QS line is a VR line. Even though it is very clear that the VR line is not the distance from point V to the QS line, this is because the VR line is not on the same plane as the QS line. As a result, the student had difficulty solving questions number 5b and 5c. This shows that the student does not understand the concept of point-to-line distance.

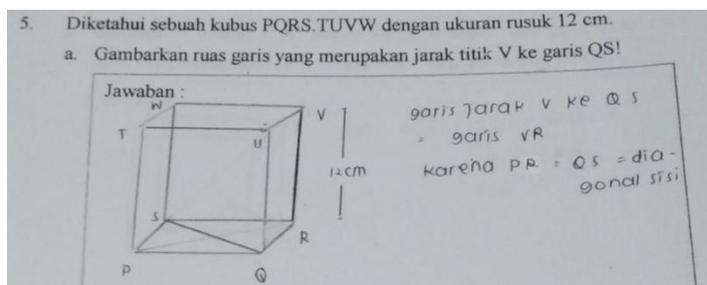


Figure 5. Students' Answers Related to Problem Number 5a.

A total of 9 students experienced obstacles in solving problem number 6a. This is because these students have not been able to determine the distance from the point to the plane. As a result, students have difficulty solving problems number 6b and 6c. The student's explanation during the interview regarding his answer to number 6c shows that the student does not understand the concept of distance.

Based on the explanation above, overall students do not experience obstacles in completing the prerequisite material. But obstacles began to be experienced by students when working on distance-in-space problems based on spatial ability indicators, especially in constructing and representing geometric models from space into flat figures. This is in line with the research of Šipuš & Čižmešija (2012) that students do not recognize the spatial form of the given object. It is also supported by Güven & Kosa (2008) that one of the causes of students' low spatial ability is presenting three-dimensional spatial information in a two-dimensional format. After conducting interviews, most students answered that they had forgotten the material on distance concepts of solid geometry, which means that most students only memorize and then forget the concept of the material. This is in line with the theory of constructivism regarding meaningful learning, according to Daryanto & Karim (2017), the process of meaningful learning is not just memorizing concepts or facts but is an activity of connecting concepts to produce a complete understanding so that the concepts learned are well understood and not easily forgotten. Then some of them also experienced obstacles in illustrating the problem in question so they were wrong in the solution. In problems with indicators of determining the actual size of the visual stimulus object, students also experience obstacles in completing the arithmetic operation, even though the concept is correct. This is because students are less careful, another reason is that the time given is not enough.

Conclusion

Based on the results and discussion previously described, it can be concluded that overall the ontogenic obstacles found in this study include psychological, instrumental, and conceptual ontogenic obstacles. First is psychological ontogenic obstacles that occur when students do not like mathematics, so they easily forget the material they have learned. Second is instrumental ontogenic obstacles that occur when students are less prepared in understanding the distance concepts of solid geometry such as distance between point-to-point, point-to-line, and point-to-plane so students are mistaken in solving solid geometry problems based on spatial ability indicators. Third is conceptual ontogenic obstacles that occur when some students do not understand arithmetic operations, so they are confused in solving arithmetic operations related to the problem. Then overall, the ontogenic obstacles that are most often experienced by students include constructing solid figures into the appropriate plane figures, constructing plane figures that are as expected, and completing arithmetic operations.

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Declaration of Interest Statement

The authors declare that they have no conflict of interests.

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